

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

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1. AGENCY USE ONLY (Leave blank)			2. REPORT DATE 12/22/94	3. REPORT TYPE AND DATES COVERED Final Jan 91 - 30 Sep 94
4. TITLE AND SUBTITLE Finite Element Technology for Penetration Problems			5. FUNDING NUMBERS DAAL03-91-G-0016	
6. AUTHOR(S) Ted Belytschko and Wing Kam Liu			8. PERFORMING ORGANIZATION REPORT NUMBER S E D T I C ELECTE FEB 08 1995 G	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Northwestern University Technological Institute Dept. of Mechanical Engineering 2145 Sheridan Road Evanston, IL 60208-3111			9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709-2211	
10. SPONSORING/MONITORING AGENCY REPORT NUMBER ARO 28320.8-MA			11. SUPPLEMENTARY NOTES The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.	
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) Finite element methods for penetration mechanics are developed. A pinball contact-impact algorithm which is easily vectorizable has been implemented on partitioned memory SIMD computers. The pinball algorithm is further extended to problems with friction and erosion; Lagrange and augmented Lagrange multiplier methods, and its convergence have also been studied. Multiple-quadrature elements with hourglass control and physical stabilization and multi-time step integration have also been studied. Numerical results of the multiple-quadrature elements with stabilization showed that simple stabilization forces can be obtained which are convergent and based on physical parameters. The implementation of this algorithm on massively parallel machines has also been investigated.				
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14. SUBJECT TERMS Finite element methods, penetration mechanics, pinball contact-impact algorithm, partitioned memory SIMD computers, friction and erosion, Lagrange and augmented Lagrange multiplier methods, Multiple-quadrature elements, hourglass control, stabilization			15. NUMBER OF PAGES 9	
16. PRICE CODE				
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

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Final Report on
FINITE ELEMENT TECHNOLOGY FOR PENETRATION PROBLEMS

Supported by the U. S. Army Research Office
Mathematical and Computer Sciences Division

Northwestern University
Department of Mechanical Engineering
Evanston Illinois 60208

January 1, 1991 to September 30, 1994

Proposal Number 28320-MA
ARO Grant DAAL03-91-G-0016

Investigators: Ted Belytschko and Wing Kam Liu

December 22, 1994

Accesion For	
NTIS	CRA&I <input checked="" type="checkbox"/>
DTIC	TAB <input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification _____	
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A. Statement of the Problem Studied

Finite element methods for penetration mechanics are developed. A pinball contact-impact algorithm which is easily vectorizable has been implemented on partitioned memory SIMD computers. Multiple-quadrature elements with hourglass control and stabilization and multi-time step integration have also been studied. The essential feature of the pinball contact-impact algorithm is to treat the impenetrability condition between bodies and the gap between bodies in contact in terms of the interpenetration of spheres embedded in the elements of the respective bodies. The pinball algorithm is further extended to problems with friction and erosion; Lagrange and augmented Lagrange multiplier methods, and its convergence have also been studied. Numerical results of the multiple-quadrature elements with stabilization showed that simple stabilization forces can be obtained which are convergent and based on physical parameters. The implementation of this algorithm on massively parallel machines has also been investigated.

B. Summary of the Most Important Results

New multiple-quadrature-point under-integrated finite elements with hourglass control are developed. The elements are selectively under-integrated to avoid volumetric and shear locking and save computational time. An approach for hourglass control is proposed such that the stabilization operators are obtained simply by taking the partial derivatives of the generalized strain rate vector with respect to the natural coordinates so that the elements require no stabilization parameter. To improve accuracy over the traditional one-point-quadrature elements, several quadrature points are used to integrate the internal forces, especially for tracing the plastic fronts in the mesh during loading and unloading in elastic-plastic analysis. Two-point and four-point quadrature elements are proposed for use in the two and three dimensional elements, respectively. Other multiple quadrature points can also be employed. Several numerical examples such as thin beam, plate and shell problems are presented to demonstrate the applicability of the proposed elements.

New element formulations based on extensions of assumed strain methods have been developed. It was found that by suppressing certain terms in the shear strains and modifying the extensional strains so that the higher order terms in the volumetric strains always vanish, the performance of the element in nearly incompressible cases, such as elastic-plastic materials, is enhanced dramatically. It was furthermore found that these assumed strain

fields could be treated either with one-point quadrature and stabilization based on the higher order terms or by 4-point quadrature without stabilization. The latter is more expensive, but it compensates for this by increased accuracy whenever elastic-plastic fronts occur in the element.

New and more stable methods for subcycling have been developed for the time integration of the equations of motion. Subcycling is crucial for efficient explicit computer solutions of many problems, especially those involving h-adaptivity, because in explicit solutions the critical time step is set by the smallest element, so a few small elements can result in order of magnitude increases in running time. With subcycling, only the smaller elements are integrated at the smaller time step, and in fact in some implementations, all elements are integrated close to their stable time step. In previous versions of subcycling, a constant velocity interpolation was used on nodes connected to elements with different time steps. In this implementation, a constant acceleration interpolation is used. Numerical studies show that it is substantially more stable. A partial proof of stability has been developed for this formulation; however, certain key steps in the stability proof still need to be resolved.

The pinball contact-impact algorithm has been extended by incorporating an automatic splitting of pinballs when the elements are too irregular to permit accurate solutions on the basis of a single pinball. This formulation has been implemented for shell elements and it has been shown that highly accurate solutions can be obtained.

We have performed an analysis of the consistency and accuracy of SPH methods and shown that consistency is not satisfied for an irregular finite SPH grid. Its accuracy is also poor in the vicinity of boundaries, as boundary conditions are not enforced effectively in current SPH methods. Since the accurate treatment of boundaries is crucial to the successful solution of problems of fracture mechanics or contact-impact, the success of SPH methods in penetration problems has been quite limited. Two gridless methods, moving least mean square interpolants and kernel functions, similar to smooth particle hydrodynamics (SPH) with correction functions, have been explored. A key feature of these methods is the introduction of a correction function on the kernel which is active primarily in the vicinities of the boundaries. We have learned that this correction function can dramatically improve the accuracy and stability of the kernel.

Introduction

Simulation of penetration and other nonlinear behavior can be of great benefit to the Army because it can improve the design of new weapons and weapon countermeasures. However, at the present time, these simulations often require as much as 100 hours of supercomputer time and still lack sufficient resolution of the important physical phenomena, which impairs their usefulness. This research is aimed at improving finite elements for penetration mechanics both from the viewpoints of speed and accuracy, the development of improved contact-impact algorithm and the implementation of these procedures on massively parallel computers.

Technical Objectives and Approach

The purpose of this project is to improve finite element methods for highly nonlinear problems such as penetration problems with particular emphasis on element stability and accuracy, contact-impact algorithms and parallelization of these algorithms. In addition, work has been started on gridless methods since they appear very promising in dealing with problems of penetration and failure.

Two new formulations of the 8-node hexahedral element for nonlinear analysis have been developed. Both forms share the characteristics that they are more stable than existing elements and do not require user-input of stabilization parameters. As a consequence, they permit efficient and accurate computation of three-dimensional penetration problems. These computations, when performed with tetrahedral elements, are often limited in resolution because they require as much as a 100 hours of supercomputer time. Compared to tetrahedral, these new elements are significantly faster and they retain the robustness of fully integrated elements.

Work is progressing on the implementation of these methods on the massively-parallel computer CM5. We have now developed a MIMD version of the exchange algorithm originally developed for the CM2. This algorithm halves the number of communications required during a time step, so it significantly speeds up the algorithm. Although our initial implementations were quite slow because they used message-passing, our latest implementation uses the gather operation in the CM5 to perform the exchange. The resulting algorithm is significantly faster than algorithms requiring both a gather and a scatter.

Our recent work has focused on gridless or meshless methods. A key breakthrough in this work has been our discovery that gridless methods such as smooth particle methods can be viewed as collocation methods based on moving least square interpolants. These constructs then enable us to overcome the stability and accuracy problems which have plagued SPH methods. At the present time, these stabilizing procedures are still quite expensive, but several promising avenues to improving their efficiency are under investigation.

C. List of All Publications and Technical Reports

Publications

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Hu, Y. K., and Liu, W. K., "An ALE Hydrodynamic Lubricated Finite Element Method with Application to Strip Rolling," *International Journal of Numerical Methods for Engineering*, Vol. 36, pp. 855-880, 1993.

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INVITED PAPER: Liu, W. K., "ALE Hydrodynamic Lubrication Finite Element Method for Strip Rolling," presented at the 4th Int. Conf. on Num. Meth. in Ind. Forming Processes, NUMIFORM92, Sophia Antipolis, September 14-18, 1992.

KEYNOTE: Liu, W. K., "Arbitrary Lagrangian-Eulerian Finite Elements for Fluid-Shell Interaction Problems" to be presented at the 7th Brazilizn Symposium on Piping and Pressure Vessels, Florianopolis, Brazil Oct 7-9, 1992.

INVITED PAPER: ARO Conference, Liu, W., K., "Multiple Quadrature Point Finite Elements," Pittsburgh, June 7-9, 1993.

INVITED PAPER: Liu, W., K., "ALE and Free Lagrangian Methods in Plasticity Calculations." Annual Meeting of Society for Industrial and Applied Mathematics, Philadelphia, Pa, July 10-13, 1993.

INVITED PAPER: Liu, W., K., "Multiple Quadrature ALE Finite Elements for Interaction Problems" at the National Taiwan University and Tsinghua University, Taiwan, August 8-12, 1993.

INVITED PAPER: Liu, W., K., "Hydrodynamic Lubricated Friction Model for Metal Forming" U. S. Association for Computational Mechanics August 16-18, 1993.

INVITED PAPER: Liu, W., K., "Multiple Scale Reproducing Kernel Methods" U. S. Association for Computational Mechanics August 16-18, 1993.

KEYNOTE: Liu, W., K., "Multiple Scale Reproducing Kernel and Wavelet Methods," *3rd World Congress for Computational Mechanics*, Chiba, Japan, August 1-5, 1994.

INVITED PAPER: T. Belytschko and W. K. Liu, "Failure of Structures subjected to Severe Underwater Environments" ONR Workshop on Underwater Explosion Effects on Structures and Shock Mitigation, University of Maryland, Sept.21-22, 1992, College Park, Maryland.

INVITED PAPER: T. Belytschko, "Adaptive Methods for Nonlinear Structural Dynamics and Crashworthiness Analysis", NASA Workshop on Computational Methods for Crashworthiness, Sept.2-3, NASA Langley.

INVITED PAPER: T. Belytschko, "Adaptivity in Nonlinear Structural Dynamics with Contact-impact" Symposium on Adaptive, Multilevel, and Hierarchical Computational Strategies, ASME Winter Annual Meeting, November 8-13, 1992, Anaheim, CA.

KEYNOTE: T. Belytschko, "Research Trends in Nonlinear Finite Element Methods for Solids and Structures" 1993 International High-Performance Computing Conference and Exhibition, April 19-22, 1993, Hsinchu, Taiwan.

INVITED PAPER: T. Belytschko, "Computational Methods and Solutions for Problems with Material Instabilities" IUTAM Symposium on Computational Mechanics of Materials, June 15-18, 1993, Brown University.

KEYNOTE: T. Belytschko, "Adaptive and Element-free Galerkin Methods for Fracture and other Failure Modes" Second U.S. National Congress on Computational Mechanics, August 16-18, 1993, Washington, D.C.

KEYNOTE: T. Belytschko, "Computational Methods for contact-impact" Impact IV, August 23-24, Berlin, Germany.

INVITED PAPER: T. Belytschko, "Adaptive Methods for Fracture Problems" Institute for Mathematics and its Applications Workshop, July 1-31, 1993, Minneapolis, Minnesota.

INVITED PAPER. "New Trends in High Resolution Algorithms", USA/ France Workshop on Numerical Simulation of Metal Forming Processes, Sophia Antipolis, France, September 27-29, 1993.

INVITED PAPER. "Parallel Computations of Shear Band Formation by Finite Elements" Symposium on Parallel Finite Element Computations, Supercomputer Institute, University of Minnesota, October 24-27, 1993.

INVITED PAPER. "Adaptive and Element Free Methods", International Symposium on Highly Advanced Computing, Sendai, Japan, November 9-11, 1993.

INVITED PAPER. "Element-free Galerkin Methods for Dynamic Fracture", NSF/ONR Symposium on Dynamic Failure Mechanisms of Modern Materials, Cal Tech, Pasadena, CA, February 2-6, 1994.

KEYNOTE. "Element Free Galerkin Methods for Static and Dynamic Crack Propagation", 3rd World Congress for Computational Mechanics, Tokyo, Japan, August 1-5, 1994.

D. List of All Participating Scientific Personnel Showing any Advanced Degrees Earned by Them While Employed on the Project

Lee P. Bindeman, Ph. D., June 1992

I.S. Yeh, Ph. D., September 1992

H.Y. Chiang, Ph.D., June 1993

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Report of Inventions

None.